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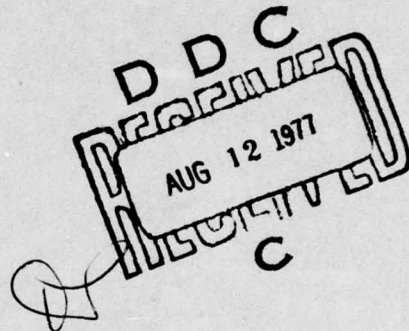
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A MATHEMATICAL PROGRAMMING MODEL FOR AN AIRCRAFT MODIFICATION PROGRAM

Dr. D. Frank Fox

August 1977

Final Report



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US ARMY AVIATION RESEARCH AND DEVELOPMENT COMMAND

Plans and Analysis Directorate

Systems and Cost Analysis Division

Developmental Systems Analysis Branch

P.O. Box 209

St. Louis, Missouri 63166

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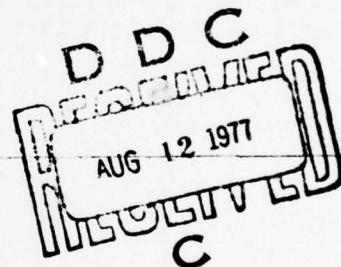
REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER USAAVRADCOM TR 77-1	2. GOVT ACCESSION NO. (12) TR-77-1	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A MATHEMATICAL PROGRAMMING MODEL FOR AN AIRCRAFT MODIFICATION PROGRAM.	5. TYPE OF REPORT & PERIOD COVERED Final Report	6. PERFORMING ORG. REPORT NUMBER DRDAV-BC-77-1
7. AUTHOR(s) Dr. D. Frank/Fox	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Aviation Research and Development Command Dir for Plans & Analysis, Systems & Cost Analysis Division, Developmental Systems Analysis Branch (DRDAV-BCA) P.O. Box 209, St. Louis, MO 63166	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Aviation Research & Development Command Directorate for Plans & Analysis, Systems & Cost Analysis Division (DRDAV-BC) P.O. Box 209, St. Louis, MO 63166	12. REPORT DATE August 1977	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 34p.	13. NUMBER OF PAGES 30	
15. SECURITY CLASS. (of this report) Unclassified		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Mathematical Model Integer Programming Linear Programming Aircraft Modification		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A large fleet of aircraft is to be modified by the installation on each aircraft in the fleet of a certain number of engineering change proposal (ECP) kits. The aircraft are deployed in smaller sub-fleets, called field units, at various locations around the world, and the kit installations are to take place at a single contractor facility. Therefore, each of the aircraft must be taken out of operation in the field and sent in to the contractor facility for modification. NEXT PAGE		

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Each field unit has an authorized strength of aircraft with a specified operational readiness to maintain, permitting only a certain number of aircraft to be away from each field unit at a given time. Furthermore, ECP kits become available over a period of time. At the beginning of the program only certain types of kits are available, and the last type of kit doesn't become available until some time later. Therefore, an aircraft sent in for modification early in the program will not get all of the ECP kits and must be sent back at least a second time.

In order to maintain operational readiness, the aircraft should be sent in as few times as possible and brought back as quickly as possible. On the other hand, aircraft can be allowed to wait at the contractor facility until additional ECP kits become available and can be installed. Hence, there are reasons for bringing the aircraft back from the contractor facility as quickly as possible and also reasons for leaving them there for extra periods of time.

An optimum solution to the problem of how many aircraft should be sent in and for how long they should remain can be developed using mathematical programming. In this paper an integer programming model consisting of constraints and an objective function is developed, making it possible to maximize the number of kits installed on the fleet of aircraft in a given period of time. The solution obtained from the model permits a kit installation schedule to be developed. The schedule will give for each field unit the number of aircraft to be sent in each month and the month in which they will be returned to the field unit. The model has application to scheduling problems of the type described above and will be useful in the solution of similar problems.

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EXECUTIVE SUMMARY

Background: The study originated as a possible means of determining an optimal method of carrying out an engineering change proposal (ECP) program on the AH-1 fleet. Later as the plans for the AH-1 program were formalized, it became clear that the model would not correspond closely enough to that program to be used for it. However, since the model employed a classical technique of operations research and would be of interest to other operations research analysts, it was decided to complete its development for possible application to similar problems.

Purpose: The purpose of the model is to find the most efficient way of carrying out a large complicated ECP program in which aircraft are sent in from various field units for modification at a single contractor facility. Different types of ECP kits become available over a period of time, and sometimes it is desirable to have the aircraft wait at the contractor facility until additional types kits become available. However, it is also desired to have the aircraft away from the field unit as little as possible. The model resolves the question of how many aircraft should be sent in each month and how long they should stay at the contractor facility.

Discussion: Although it was decided not to use the model for the AH-1 program, the model has been successfully used on a sample problem using hypothetical data.

Conclusions: For the situation described in the Introduction and Assumptions sections, a valid workable model has been constructed. The model may be varied by using only some of the constraints or by selecting from different objective functions. In carrying out an ECP program it is possible to maximize the number of aircraft modified. The solution of the model provides a schedule by field unit of how many aircraft are to be sent in for modification each month and the month in which they are to return to the field unit.

Recommendations: The model should be used for programs which conform to it or to which it can be adapted.

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A MATHEMATICAL PROGRAMMING MODEL FOR AN AIRCRAFT MODIFICATION PROGRAM

1. INTRODUCTION

A large fleet of aircraft is to be modified by the installation on each aircraft in the fleet of a certain number of engineering change proposal (ECP) kits. The aircraft are deployed in smaller sub-fleets, called field units, at various locations around the world; and the kit installations are to take place at a single contractor facility. Therefore, each of the aircraft must be taken out of operation in the field and sent in to the contractor facility for modification.

Each field unit has an authorized strength of aircraft with a specified operational readiness to maintain, permitting only a certain number of aircraft to be away from each field unit at a given time. Furthermore, ECP kits become available over a period of time. At the beginning of the program only a certain type of kits are available, and the last type of kit doesn't become available until some time later. Therefore, an aircraft sent in for modification early in the program will not get all of the ECP kits and must be sent back at least a second time.

In order to maintain operational readiness, the aircraft should be sent in as few times as possible and brought back as quickly as possible. On the other hand, if an aircraft is allowed to wait at the contractor facility until additional ECP kits become available and can be installed,

it can be returned to the field unit with more modifications, making it better able to perform its mission and perhaps also eliminating a future trip to the contractor facility for those modifications for which it waited. Hence, there are reasons for bringing the aircraft back from the contractor facility as quickly as possible and also reasons for leaving them there for extra periods of time.

An optimum solution to the problem of how many aircraft should be sent in and for how long they should remain can be developed using mathematical programming. In this paper an integer programming model consisting of constraints and an objective function is developed, making it possible to maximize the number of kits installed on the fleet of aircraft in a given period of time. The solution obtained from the model permits a kit installation schedule to be developed. The schedule will give for each field unit the number of aircraft to be sent in each month and the month in which they will be returned to the field unit. The model has application to scheduling problems of the type described above and will be useful in the solution of similar problems.

2. SUMMARY OF CONSTRAINTS

Since the full description of the constraints is rather lengthy and complicated, a summary of them in descriptive form is given here for the convenience of the reader. The constraints are applicable in the context described in the Introduction and Applications sections. A detailed description of the constraints and their mathematical representations can be found later in the report in the appropriate sections.

Aircraft Availability Constraint: No more aircraft are selected for processing from a field unit than there are aircraft at that field unit.

Operational Readiness Constraint: For each field unit there is a maximum number of aircraft which can be away at any given time.

Contractor Facility Capacity Constraint: There is a maximum number of aircraft which can be at the contractor facility in a given month.

Application Constraint: At each field unit there is a minimum number of aircraft which will have some kits installed.

Objective Function: (1) Maximize the number of aircraft modified or
(2) Maximize the number of kits installed.

3. ASSUMPTIONS

When constructing a model it is desired to model the actual physical situation as closely as possible. However, it is often impossible to model all aspects of a situation in complete detail. Certain assumptions were made here in order to include as much complexity as possible while still keeping the model manageable.

The kit installation program will run for a certain number of time periods. A time period can be any fixed length of time such as a month, quarter, year, etc. For convenience of expression let the time periods be months and let the length of the program be T months.

First it is assumed that at the beginning of the program all of the aircraft are in need of all the kits. Also it is assumed that the kits are numbered according to availability; the first type of kit available is kit number 1, the second type of kit available is kit number 2, etc. Further it is assumed that for each type of kit there is a month in which that kit becomes available in whatever quantities it is needed from then on.

In practice the monthly demand for kits is probably uniform enough to make this a reasonable assumption.

Hence, the following table of kit availability can be constructed.

TABLE 1

TABLE OF KIT AVAILABILITY

MONTH	1	2	3	4	. . .	T
KIT	K_1	K_2	K_3	K_4	. . .	K_T

The table has the following interpretation. All aircraft shipped from the contractor facility in the 1st month will have received kits 1 through K_1 , those shipped in the 2nd month will have received kits 1 through K_2 , those shipped in the 3rd month will have received kits 1 through K_3 , etc. If all the kits become available before the T^{th} month, there will be some month c such that from the c^{th} month on all aircraft shipped from the contractor facility will have all kits.

Implicit in this assumption is the assumption that while an aircraft is at the contractor facility, it receives all of the kits which are available before it is shipped back. Furthermore, it is assumed that the installation time for all groups of kits is the same, no matter how many kits there are in a group.

In order to decrease the number of possibilities represented by the variables it is assumed that an aircraft will be sent in no more than twice. Also no aircraft at a field unit will be sent in the second time until every aircraft at that field unit has been sent in once. Furthermore, no aircraft will be sent in the second time until it can have all of its remaining

needed kits installed without any waiting time (in other words until the earliest month in which it would be shipped from the contractor facility is the c^{th} month or later).

Let

X_{ijk} = the number of aircraft sent in for the first time from the i^{th} field unit in the j^{th} month which are shipped back from the contractor facility in the k^{th} month, and let

Y_{ijk} = the number of aircraft sent in for the second time from the i^{th} field unit in the j^{th} month which are shipped back from the contractor facility in the k^{th} month.

The subscripts j and k are used to indicate the month j in which the aircraft are sent in for modification and the month k in which the aircraft are shipped back from the contractor facility. A given subscript j may have many associated values of k which in effect allows the aircraft to stay at the contractor facility for various lengths of time, permitting it to wait at the contractor facility until additional kits become available and can be installed.

4. PERMISSIBLE VARIABLES

In the variable X_{ijk} , the subscript j represents the month in which the aircraft are shipped to the contractor facility, and the subscript k represents the month in which the aircraft are shipped from the contractor facility. Therefore, the value of j must always be less than the value of k , limiting the possibilities for subscripts to certain values.

Let S_i be the transportation time for an aircraft to be shipped from the i^{th} field unit to the contractor facility, and assume that the transportation time for shipment back to the field unit is also S_i . Let p be the installation time for a group of kits. Recall that p is independent of the number of kits in a group.

If an aircraft is shipped in during the j^{th} month, it will arrive at the contractor facility in the $j + S_i^{\text{th}}$ month and take p months to be processed. Therefore, the first month in which it can be shipped from the contractor facility is the $j + S_i + p^{\text{th}}$ month. However, since an aircraft is allowed to wait at the contractor facility for the availability of additional kits, it may not be shipped back in the $j + S_i + p^{\text{th}}$ month. That month or any later month is a possible shipping month. Hence the variable X_{ijk} cannot have a subscript k with $k < j + S_i + p$, thereby limiting the variables which can occur to those listed in the following table.

TABLE 2

TABLE OF PERMISSIBLE SUBSCRIPTS FOR X_{ijk}

$X_{i,1,1+S_i+p}$	$X_{i,1,2+S_i+p}$	$X_{i,1,3+S_i+p}$...	$X_{i,1,T}$
	$X_{i,2,2+S_i+p}$	$X_{i,2,3+S_i+p}$...	$X_{i,2,T}$
		$X_{i,3,3+S_i+p}$...	$X_{i,3,T}$
		.		
		.		
		.		
				$X_{i,T-S_i-p,T}$

Recall that there is some month c such that all aircraft shipped from the contractor facility in the c^{th} month or a later month will have had all the kits installed. Further, it was assumed that aircraft are not sent in for a second time until all kits are available. Specifically, the aircraft which are sent in for the second time will not be sent in before the month which permits c to be the first month in which they could be shipped from the contractor facility. Since the shipping time from the i^{th} field unit is S_i and the processing time is p , in the variable Y_{ijk} it is necessary that $j \geq c - S_i - p$. Furthermore, since an aircraft sent in for the second time receives all the remaining needed kits, there is no need for it to stay at the contractor facility longer than the p months necessary to install the kits; in other words, there is no waiting

time for additional kits. Therefore, the subscripts on Y_{ijk} are limited to those listed in the following table.

TABLE 3

TABLE OF PERMISSIBLE SUBSCRIPTS FOR Y_{ijk}

$Y_{i,c-S_i-p,c}$	$Y_{i,c-S_i-p+1,c+1}$...	$Y_{i,T-S_i-p,T}$
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An alternative to having a list of permitted subscripts would be to allow all subscripts to be used and to impose the constraint that the inadmissible variables have the value zero. However, this method was not used in order to keep the number of variables and constraints used to a minimum.

5. AIRCRAFT AVAILABILITY CONSTRAINT

At each field unit there are a certain number of aircraft. In essence, the aircraft availability constraint prevents more aircraft from being sent in from a given field unit than there are aircraft at that field unit. However in applying this constraint, it is not satisfactory to simply count the number of aircraft sent in, because some aircraft are sent in twice. Therefore, it is necessary to have two separate constraints, one for those aircraft being sent in for the first time and one for those being sent in for the second time.

Let N_i = the number of aircraft stationed at the i^{th} field unit at the start of the modification program. For each field unit, the total number of aircraft sent in for the first time during the T months of the program must be less than or equal to N_i . As an inequality the constraint is written

$$\sum_j \sum_k x_{ijk} \leq N_i, \quad (1a)$$

for each field unit i . The left-hand side of (1a) represents the total number of aircraft sent in for the first time during the T months of the program. In order to avoid cumbersome expressions in the limits of the summation notation, the limits are sometimes omitted. If no limits are given in a summation, it is to be understood that the sum is over all permissible values of the subscript. For example $\sum_j \sum_k x_{ijk}$ represents the sum over all permissible subscript j and k for x_{ijk} , in other words over all values listed in Table 2.

If all the kits become available for installation before the T^{th} month, there will be some month c such that all aircraft shipped from the contractor facility in the c^{th} month or a later month will have all the kits installed. In other words, any group X_{ijk} of a aircraft with $k \geq c$ will have all the kits installed the first time they are sent in and will not go in a second time. Therefore, not all the aircraft are possibilities for being sent in a second time, and the number of aircraft available to be sent in a second time must be decreased by the number which have all the kits installed the first time. The number of aircraft from the i^{th} field unit which have all the kits installed the first time is

$$\sum_j \sum_k X_{ijk} ,$$

Hence the constraint is written as

$$\sum_j \sum_k Y_{ijk} \leq N_i - \sum_j \sum_k X_{ijk} , \quad (1b)$$

for each field unit i .

6. OPERATIONAL READINESS CONSTRAINT

In each field unit some of the aircraft can be away from the field unit for ECP kit installation. However, in order to maintain the required operational readiness of the fleet, there is a limit on the number of aircraft which can be away at any given time.

Let M_i = the number of aircraft which can be away from the i^{th} field unit for ECP kit installation when there are N_i aircraft assigned to the field unit. The operational readiness constraint is that the number of aircraft away from the i^{th} field unit in each month must be less than or equal to M_i .

An expression is needed for the number of aircraft which are away from the i^{th} field unit during the r^{th} month. Those which will be away during the r^{th} month are those which are sent in during the r^{th} month or earlier and which have not returned by the r^{th} month. In order to arrive back at the field unit in the r^{th} month, an aircraft must be shipped from the contractor facility in the $r - S_i^{\text{th}}$ month. Hence, for the variable X_{ijk} if $k > r - S_i$, the aircraft will not have returned to the field unit by the r^{th} month.

The number of aircraft away from the i^{th} field unit during the r^{th} month will first be established for the variables X_{ijk} by month for the months in which they are sent in from the field unit. The number away from the i^{th} field unit in the r^{th} month which were sent in during the

1st month is

$$X_{i,1,q_1} + X_{i,1,q_1+1} + X_{i,1,q_1+2} + \dots + X_{i,1,T} = \sum_{k=q_1}^T X_{i1k}$$

where q_1 is the larger of the two numbers $r - S_i + 1$ and $1 + S_i + p$. It is necessary that $q_1 \geq r - S_i + 1$ so that the aircraft will not have returned to the field unit by the r^{th} month, and it is necessary that $q_1 \geq 1 + S_i + p$ because $S_i + p$ is the minimum time for aircraft to be sent in and have kits installed.

The number away from the i^{th} field unit in the r^{th} month which were sent in during the 2nd month is

$$X_{i,2,q_2} + X_{i,2,q_2+1} + X_{i,2,q_2+2} + \dots + X_{i,2,T} = \sum_{k=q_2}^T X_{i2k}$$

where q_2 is the larger of $r - S_i + 1$ and $2 + S_i + p$.

Similar expressions can be written for the 3rd, 4th, ..., $r-1^{\text{st}}$ months.

The number away from the i^{th} field unit during the r^{th} month which were sent in during the r^{th} is

$$X_{i,r,q_r} + X_{i,r,q_r+1} + X_{i,r,q_r+2} + \dots + X_{i,r,T} = \sum_{k=q_r}^T X_{irk}$$

where $q_r = r + S_i + p$.

For the variables Y_{ijk} , the number of aircraft sent in from the i th field unit before the r th month which have not returned by the r th month is

$$Y_{i,q,q+S_i+p} + Y_{i,q+1,q+S_i+p+1} + \dots + Y_{i,r,r+S_i+p} = \sum_{j=q}^r Y_{i,j,j+S_i+p}$$

where q is the larger of $r - 2S_i - p + 1$ and $c - S_i - p$. It is necessary that $q \geq r - 2S_i - p + 1$ so that the aircraft will not have returned to the field unit by the r th month, and it is necessary that $q \geq c - S_i - p$, since $c - S_i - p$ is the smallest permissible value of j for Y_{ijk} .

Finally the constraint can be written as

$$\sum_{k=q_1}^T X_{i1k} + \sum_{k=q_2}^T X_{i2k} + \dots + \sum_{k=q_r}^T X_{irk} + \sum_{j=q}^r Y_{i,j,j+S_i+p} \leq M_i \quad (2)$$

for each field unit i and for each month r where

$$q_j = \max \{r - S_i + 1, j + S_i + p\} \text{ and } q = \max \{r - 2S_i - p + 1, c - S_i - p\}.$$

7. CONTRACTOR FACILITY CAPACITY CONSTRAINT

The contractor facility capacity constraint places a limitation on the number of aircraft which can be at the contractor facility in any given month. Let F_r be the maximum number of aircraft which can be at the contractor facility during the r^{th} month.

Now an expression is needed for the number of aircraft at the contractor facility during the r^{th} month. An aircraft from the i^{th} field unit will be at the contractor facility in the r^{th} month if it has been shipped in by the $r-S_i^{\text{th}}$ month so that it will have time to arrive, and if it is shipped back after the r^{th} month. In other words, if the variable X_{ijk} has $j \leq r-S_i$ and $k > r$.

For aircraft sent in for the first time the number of aircraft at the contractor facility in the r^{th} month is

$$\begin{aligned} & \sum_i (X_{i,1,r+1} + X_{i,1,r+2} + \dots + X_{i,1,T} + \\ & \quad X_{i,2,r+1} + X_{i,2,r+2} + \dots + X_{i,2,T} + \\ & \quad \vdots \\ & \quad X_{i,r-S_i,r+1} + X_{i,r-S_i,r+2} + \dots + X_{i,r-S_i,T}) = \\ & \quad \sum_i \sum_{j=1}^{r-S_i} \sum_{k=r+1}^T X_{ijk}. \end{aligned}$$

The sum on j includes all possible months in which the aircraft can be

shipped in and have arrived by the r^{th} month, namely months 1, 2, 3, ... , $r-S_i$. The sum on k includes all possible shipping months after the r^{th} , namely months $r+1$, $r+2$, ... , T . The sum on i includes all the field units. Further, the subscripts are limited to permissible values; for example $X_{i,r-S_i,r+1}$ would not occur unless $p=1$, since the smallest permissible value of k in X_{ijk} is $j+S_i+p$.

For the aircraft sent in for the second time the number which will be at the contractor facility in the r^{th} month is

$$\sum_i (Y_{i,q,q+S_i+p} + Y_{i,q+1,q+S_i+p+1} + \dots + Y_{i,r-S_i,r+p})$$

$$\sum_i \sum_{j=q}^{r-S_i} Y_{i,j,j+S_i+p}$$

where $q = \max \{r-S_i-p+1, c-S_i-p\}$. It is necessary that $q \geq c-S_i-p$, because $c-S_i-p$ is the smallest permissible value of j in Y_{ijk} ; and it is necessary that $q \geq r-S_i-p+1$ in order that the aircraft will not have been shipped back by the r^{th} month.

Hence, the constraint is written as

$$\sum_i \left[\sum_{k=r+1}^T \sum_{j=1}^{r-S_i} X_{ijk} + \sum_{j=q}^{r-S_i} Y_{i,j,j+S_i+p} \right] \leq F_r \quad (3)$$

for each month r where

$$q = \max \{r-S_i-p+1, c-S_i-p\}.$$

Recall that M_i is the number of aircraft that can be away from the i^{th} field unit in the r^{th} month. Then $\sum_i M_i$ is the total number which can be away from all the field units in the r^{th} month and hence the maximum number which could be at the contractor facility. If $\sum_i M_i \leq F_r$ for each month r , then the contractor facility capacity constraint is automatically satisfied, and in that case, it can be omitted in the formulation of the problem.

8. APPLICATION CONSTRAINT

There may be some reason why aircraft from certain field units are selected first by the model for kit installation. For example, the transportation time or cost from an overseas location might be so much greater than those for other units that in the optimization process they would not be selected. However, those overseas aircraft might be the ones whose modification was most desired. Therefore, there is an application constraint which requires that at least a certain number of aircraft from each field unit will have some kits installed.

For example the constraint might require that one-fifth of all the aircraft at each unit have some kits installed; or it might require that all the aircraft at one particular unit have some kits installed and that one-third of the aircraft at the other units have some kits installed. Care must be exercised, however, not to impose such a demanding constraint that a feasible solution is impossible.

For each field unit i , let A_i be the number of aircraft which must have at least some kits installed. Then the constraint becomes

$$\sum_{j,k} x_{ijk} \geq A_i \quad (4)$$

for each field unit i , and the summations on j and k are taken over their permissible values.

9. OBJECTIVE FUNCTION

An objective function must be selected which will cause the optimization of some feature of the modification program. Two choices which come immediately to mind are minimizing the cost of the program and minimizing the out of service time of the aircraft. However, if the cost is minimized, the optimum solution is to install no kits. Then the cost is zero and has been minimized. If the application constraint is applied to specify a minimum number of aircraft which must be modified, minimizing the cost will limit the modified aircraft to the minimum specified number. This is contrary to the desired result which is to modify as many aircraft as possible while keeping the cost to a minimum. Minimizing the out of service time produces a similar result, send in no aircraft or send in the minimum number of aircraft.

Therefore, it was decided to use the following objective function:

$$Z = \sum_{i,j,k} (A_{ijk} X_{ijk} + B_{ijk} Y_{ijk})$$

where the coefficients A_{ijk} and B_{ijk} are weight factors which don't necessarily sum to one. The relative sizes of the weights can be chosen to indicate the relative importance of having kits installed on the corresponding groups of aircraft. All that remains is to make a suitable choice for the weights. Two possible choices are discussed.

One possible choice for the weights is $A_{ijk} = B_{ijk} = 1$ for all i, j , and k . Then the objective function represents the total number of

aircraft modified where aircraft sent in twice are counted twice. Hence, it is possible to maximize the number of aircraft modified.

If $Y_{ijk} = 0$ for all i, j , and k , the objective function becomes

$\sum_{i,j,k} A_{ijk} X_{ijk}$. For each variable X_{ijk} , the weight A_{ijk} can be chosen to

be the number of kits installed on the corresponding group of aircraft. Since the number of kits installed is determined entirely by the subscript k , the coefficients A_{ijk} are obtained from the table of kit availability (Table 1). Hence, the objective function represents the number of kits installed, making it possible to maximize the number of kits installed.

If some of the Y_{ijk} are not zero, an objective function representing the number of kits installed can not always be obtained. Aircraft sent in for the second time will have all the remaining needed kits installed. However in general, before a solution to the model is obtained, there is no way of knowing how many kits the aircraft will have installed the first time they are sent in and therefore no way to know how many kits will be installed the second time. In order to obtain an objective function in this case, let A_{ijk} be the number of kits installed on the group X_{ijk} and choose B_{ijk} to be an estimation of the number of kits installed on the group Y_{ijk} . For example, if each aircraft is to have K kits installed during the modification program, B_{ijk} might be taken to be $\frac{K}{2}$ for all i, j, k . Then the objective function will be an approximation to the total number of kits which are installed.

With the objective function just described, a solution to the model can be obtained. From the solution, an improved estimate for the B_{ijk} can be

made. These new values for B_{ijk} can be used to obtain a second solution to the model. This iteration process can be continued if desired. However, there is no certainty that improvements will occur after the second solution. Furthermore, the choice of the B_{ijk} may not be critical. Therefore, it may be satisfactory to stop with the second or even the first approximation.

If the number of aircraft processed is maximized, that puts a premium on a large number of aircraft with perhaps only a few kits installed on each. Whereas maximizing the number of kits installed necessitates installing more kits on fewer aircraft. Maximizing the number of aircraft processed might result in an aircraft being sent in for only one kit. While maximizing the number of kits might result in the aircraft waiting at the contractor facility until more kits are available. Therefore, it is considered better to maximize the number of kits installed.

10. SOLUTION

The model is an integer programming model. In all but the most trivial case, the solution would be too involved to obtain manually, and therefore requires a computer procedure. Once a solution is obtained, it consists of values of X_{ijk} and Y_{ijk} for all permissible subscripts i, j, k . Recall that i designates the field unit, j the month in which the aircraft are sent in, k the month in which they are sent back, and X and Y the number of aircraft for the first for the first and second times, respectively. Therefore, a solution to the model is a list by field unit of how many aircraft are to be sent in each month for the first and second times. The list also includes the month in which they are sent back.

11. VARIATIONS

There are at least three important changes which can be made in the model as it has been described so far.

First it is possible to include a cost constraint. It is necessary to determine the cost of installing each kit and the transportation cost of shipping an aircraft from each field unit to the contractor facility. Then an expression can be obtained for the cost expended for an interval of time, for example one year. The cost constraint can then be written, limiting the expenditures in each year to a specified amount. The amount of course could vary from one year to the next. If desired, it is also possible to specify that at least a minimum amount be spent each year.

A second variation which can be made in the constraints is to allow the number of aircraft at a field unit and hence the number of aircraft which can be away from that field unit at any given time to vary. This would permit the introduction of new aircraft into the fleet or the removal of aircraft from the fleet.

The final variation is the most important. When the original model was developed, it seemed that the constraints would be so restrictive that all of the aircraft could not be modified in the specified length of the modification program. Therefore, an objective function was chosen which would maximize the number of aircraft modified or the number of kits installed.

Another approach is to use the application constraint to require that all the aircraft receive all of the modifications. In order to have a

feasible solution, some of the other constraints will undoubtedly have to be relaxed, and the total length of the program may have to be increased.

With all of the aircraft receiving all of the modifications, the old objective function is unusable. Instead the objective function can be chosen to be the total cost of the program. Moreover, since it is reasonable to assume that the costs of installing the kits is independent of how the aircraft are brought in for modification, the only variable costs in the modification program are the transportation costs. Furthermore, the only variation in the transportation cost is in the number of trips to the contractor facility. Therefore, one possible objective function is the total transportation cost of the program, and minimizing the transportation cost minimizes the number of trips to the contractor facility. Another possible objective function is the total out of service time while the aircraft are being modified. The out of service time for a group of aircraft X_{ijk} can be determined from the subscripts j and k and from the transportation time S_i . In fact the out of service time for the group X_{ijk} would be $k+S_i-j$. An expression can then be obtained for the total out of service time for all aircraft, and the program can be carried out to minimize the total out of service time.

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